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Ornament as Algebra: A Group-Theoretic Ethnomathematical Exploration of The Ceramic Symmetries of Masjid Tiban, Malang, Indonesia

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ABSTRACT

Ethnomathematics research on Indonesian architecture has largely catalogued plane figures and solid shapes, leaving the algebraic structure of cultural ornament almost unexplored. This study examines the blue-and-white ceramic ornament of Masjid Tiban, the mosque within the Bihaaru Bahri 'Asali Fadlaailir Rahmah Islamic boarding school in Turen, Malang, through a single mathematical lens: group theory, specifically the theory of symmetry groups. Using a qualitative ethnographic exploratory design combined with an etic–emic ethnomodelling perspective, the study analysed photographs of five ornament zones (main gate, information hall, residential façade, balustrade, and courtyard) and documentary sources describing a complex designed without drawings and built by hand over decades. Twelve ornament units were classified with the Washburn–Crowe flowchart and then analysed structurally through subgroup relations, Lagrange's theorem, the orbit–stabiliser theorem, and Burnside's lemma. The ornament embodies five rosette groups (C_1, D_1, C_4, D_4, D_8), three frieze groups (p1, p1m1, p2mm), two wallpaper groups (p4, p4m), and cyclic actions arising when friezes close around cylindrical drums and vessels. These groups form explicit subgroup chains of index two, so that framing, chirality, and calligraphy act as visible mechanisms of symmetry breaking. Burnside counting shows that the eight-ray star admits 30 distinct two-colour designs under D_8 but 51 under its framed D_4 setting. The study contributes a culturally grounded entry point to abstract algebra and a five-phase learning trajectory for pre-service mathematics teachers, extending ethnomathematics from shape recognition to structure.

Keywords: Ethnomathematics; Group Theory; Symmetry Groups; Frieze Patterns; Islamic Architecture

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INTRODUCTION

Ethnomathematics, as framed by D'Ambrosio (1985), studies the mathematical ideas, techniques, and ways of knowing that cultural groups develop as they count, measure, design, and build. Over four decades the field has moved from documenting "folk" procedures towards a broader claim that mathematics is a family of culturally situated practices whose meanings are negotiated between local knowledge and academic mathematics (Barton, 1996). Rosa and Orey (2016) describe this negotiation as *ethnomodelling*, in which an emic account of how a community understands its own practice is placed in dialogue with an etic, formal mathematical model. Bibliometric evidence shows that publications in the field have grown steadily and

peaked around 2020, with Indonesia among the most productive contributors (Noerhasmalina & Hidayatulloh, 2023).

Indonesian studies have documented mathematics in batik (Prahmana & D'Ambrosio, 2020), Sasak vernacular architecture (Supiyati et al., 2019), Sundanese measuring and pattern-making practices (Muhtadi et al., 2017), and Buginese food culture (Pathuddin et al., 2021). A systematic review of Sundanese ethnomathematics, however, found that most studies stop at identifying elementary geometric figures and seldom reach structured curricular use (Lidinillah et al., 2022). The same pattern appears in research on religious buildings: mosques are typically read as collections of squares, circles, cylinders, and domes. Such readings are useful for school geometry, but they treat ornament as a set of *shapes* rather than as a *system of transformations*, and they leave teachers with limited guidance on how to move beyond recognition tasks (Sunzuma & Maharaj, 2019; Verner et al., 2019).

A more powerful lens is available in group theory. Any repeating ornament is invariant under a set of isometries (rotations, reflections, translations, and glide reflections) that is closed under composition, contains the identity and inverses, and therefore forms a group. Finite ornaments are classified by the cyclic groups C_n and dihedral groups D_n; one-directional bands fall into exactly seven frieze groups; and two-directional patterns fall into exactly 17 wallpaper groups (Conway et al., 2008; Schattschneider, 1978). Washburn and Crowe (1988) turned this classification into a practical flowchart for anthropologists, and analyses of Islamic architecture have shown that artisans realised highly sophisticated symmetry structures long before they were formalised (Lu & Steinhardt, 2007). In Indonesia, frieze and wallpaper groups have been applied mainly to textiles: Siak songket (Indriati et al., 2022), Toba Batak *ulos* (Suwanto et al., 2022), East Nusa Tenggara *tenun ikat* (Nataliani, 2022), Malay Deli songket (Mingka et al., 2023), and Toba Batak *gorga* ornament (Sinaga & Kartika, 2024). Most of these studies emphasise classification or computer generation of new motifs; few examine how the groups relate to one another, and none of them addresses three-dimensional architectural ceramics.

Masjid Tiban in Sananrejo, Turen, Malang Regency, East Java, offers an unusually rich case. The mosque is part of the Bihaaru Bahri 'Asali Fadlaailir Rahmah Islamic boarding school (*pondok pesantren*), which grew from the family home of Romo Kyai Ahmad Bahru Mafdlaluddin Shaleh Al-Mahbub Rahmat Alam in 1963 and became a formal pesantren in the late 1970s (Hakim, 2016; Sholah, 2018). The ten-storey complex was designed by the founder himself, without formal architectural training or construction drawings and reportedly guided by *istikharah* prayer, and it was built gradually by hand by students, alumni, and congregants (AW, 2014; Sholah, 2018; Suryo Putro et al., 2023). Its surfaces are almost entirely clad in blue-and-white glazed ceramic tiles and moulded relief. A recent study explored plane figures, solid figures, and geometric transformations in the mosque's architecture (Sholikhah et al., 2025), but it described transformations individually and did not examine the algebraic structure formed by the symmetries taken together.

This gap matters for mathematics education as well as for cultural documentation. Group theory is a gateway course for prospective mathematics teachers, yet students struggle with the abstract notions of group, subgroup, coset, and action (Dubinsky et al., 1994; Melhuish, 2019). Instructional research shows that these notions can be guided towards reinvention by starting from the symmetries of concrete figures (Larsen, 2013). Culturally meaningful ornament could serve as that starting point, provided its group structure is first established rigorously. Accordingly, this study adopts a single mathematical topic, group theory of symmetry, and pursues three objectives: (1) to identify the symmetry groups embodied in the ceramic ornament of Masjid Tiban; (2) to analyse how these groups are structurally related through

subgroups, group actions, and counting; and (3) to derive an integration pathway for teaching introductory group theory. The central argument is that the ornament of Masjid Tiban is not merely decorated with shapes but organised as a coherent hierarchy of symmetry groups, and that this hierarchy makes abstract algebraic structure visible.

METHOD

This section describes the research design, the site and data sources, the data collection procedure, and the analytical protocol used to move from photographs of ornament to group-theoretic structure.

Research Design

The study used a qualitative ethnographic exploratory design (Creswell & Poth, 2018) organised by the ethnomodelling perspective of Rosa and Orey (2016). The *emic* strand concerns how the ornament came into being and what it means within the pesantren community, as recorded in documentary sources. The *etic* strand constructs a formal model of the same ornament using the classification of symmetry groups. A *dialogic* step then compares the two strands and translates the resulting model into a learning pathway. The study is therefore both exploratory (documenting mathematics in a cultural artefact) and integrative (proposing its use in mathematics instruction).

Research Site, Units of Analysis, and Sources

The research site was the Masjid Tiban complex in Sananrejo Village, Turen District, Malang Regency, East Java, Indonesia. Ornament units were selected purposively according to three criteria: (a) the unit is produced by deliberate repetition of a motif, so that a symmetry group can be defined; (b) the unit is documented clearly enough to identify its fundamental motif and repeat; and (c) the selection, as a whole, spans the three scales at which symmetry operates, namely single tiles (rosette level), bands (frieze level), and surfaces (wallpaper level), together with curved surfaces. Emic information was drawn from written sources produced within or close to the pesantren community: a historical study of the pesantren (Hakim, 2016), an ethnographic study of its santri and ideology (Sholah, 2018), an architectural critique of the building (Suryo Putro et al., 2023), and a journalistic account based on visits to the complex (AW, 2014).

Data Collection

Data were collected in two forms:

(1) **Visual documentation.** Photographic documentation of five ornament zones: the main gate, the information hall (*ruang informasi*), the multi-storey residential façade, the balustrade and arched gables of the upper terraces, and the entrance courtyard with its tower and monumental vessel (Figures 1–5). Two photographs are field documentation and three were obtained from published media sources with permission. Each ornament unit was examined in at least three repeats to separate the designed pattern from local damage or irregular installation.

(2) **Documentary sources.** Four written sources on the pesantren's founding, its design process, and its construction practices (AW, 2014; Hakim, 2016; Sholah, 2018; Suryo Putro et al., 2023). Statements relevant to design, labour, materials, and ornament were extracted, compared across sources, and coded into emic themes; where sources disagreed (for example, on dates), the disagreement was recorded rather than resolved.

Data Analysis Techniques

Analysis followed a five-step protocol:

(1) **Unit identification.** For each ornament, the smallest motif whose repetition generates the whole (the fundamental motif) and the translation directions were identified.

(2) **Classification.** The Washburn Crowe flowchart (Washburn & Crowe, 1988) was applied: a pattern without translations was assigned a rosette group C_n or D_n; a pattern with one translation direction was tested for half-turns, transverse mirrors, longitudinal mirrors, and glide reflections to assign one of the seven frieze groups; a pattern with two independent translations was tested for the highest rotation order, mirrors, and glides to assign one of the 17 wallpaper groups. Groups are reported in IUC notation with Conway orbifold notation in parentheses (Conway et al., 2008).

(3) **Idealisation.** Classification concerned the designed pattern. Perspective distortion, lighting, weathering, and minor installation errors were disregarded, whereas glaze colour and moulded infill were treated as part of the design because they are deliberate.

(4) **Structural analysis.** Relations among the identified groups were examined through subgroup inclusion and index (Lagrange's theorem), the orbit stabiliser theorem for the action of a group on motif elements, and Burnside's lemma for counting design variants. All counts were verified by exhaustive computer enumeration.

(5) **Integration.** The verified structures were mapped onto the conceptual sequence of an introductory group theory course, following the guided-reinvention approach of Larsen (2013).

RESULTS AND DISCUSSION

Results

Emic Context: How the Ornament Came Into Being

The documentary sources agree on the main lines of the complex's history while differing on several dates. The founder, born in Sananrejo in 1943 (Sholah, 2018), began receiving students in his family home in 1963; the pesantren is dated as formally established in 1976 (Suryo Putro et al., 2023) or 1978 (Hakim, 2016). Construction continued in phases for decades: it was halted in the early 1990s, when local authorities required a building permit, and resumed in 1998 (AW, 2014; Suryo Putro et al., 2023). By 2010 the main building had reached nine storeys (Hakim, 2016), and it now has ten, standing on a site of about four hectares of which roughly 1.5 hectares are built (Suryo Putro et al., 2023).

Three emic themes bear directly on the ornament. The first is *design without drawings*. The founder had no architectural or engineering training, no formal drawings were produced, and the form of the building is attributed to *istikharah* prayer and to an inner method he called *siri* (AW, 2014; Hakim, 2016; Suryo Putro et al., 2023). The second is *collective manual labour*. Resident santri, alumni, and congregants built the complex themselves with simple tools and without heavy machinery, starting from basic materials such as red brick bonded with clay, and its expansion is described as following the wishes of the congregation rather than a fixed master plan (Sholah, 2018; Suryo Putro et al., 2023). The third is *deliberate eclecticism under a unified palette*. The sources describe a blend of Middle Eastern, Chinese, and Indian forms, held together by a dominant blue-and-white scheme of ceramic mosaics and by calligraphy of Qur'anic verses and the ninety-nine names of God (*Asmaul Husna*) (AW, 2014; Suryo Putro et al., 2023). These themes frame the etic analysis that follows: ornament produced without a global

blueprint, by many hands, over many years, nonetheless displays a consistent symmetry grammar.

Ornament Zones of Masjid Tiban

Before turning to the mathematics, this subsection presents the five ornament zones as they appear on site. The main gate (Figure 1) is a monumental portal whose pillars are clad from base to cornice with square star tiles framed by moulded vine borders; above the opening rise a tiled drum with an arcaded gallery and two calligraphic panels.



Figure 1. Main gate of the Masjid Tiban complex, Turen, Malang

The information hall (Figure 2) shows the densest interior cladding: columns of pyramidal relief tiles, cornices of scrolled cartouches, a diagonal lattice screen, and fields of square-meander tiles. On the residential façade (Figure 3), walls and columns are covered with a diagonal chequer of blue squares set in a white lattice.



Figure 2. Information hall clad in blue-and-white ceramic ornament



Figure 3. Residential façade with diagonal blue–white chequer cladding on walls and columns

The upper terraces (Figure 4) repeat arched gables, globe-topped posts, and a lower arcade of small tiled arches, while the entrance courtyard (Figure 5) gathers the tiered tower, pillars of star tiles, and a monumental egg-shaped vessel wrapped in horizontal ornamental bands.



Figure 4. Arched gables, balustrade, and canopies of the upper terraces



Figure 5. Entrance courtyard with the tiered tower, tiled pillars, and the monumental egg-shaped vessel

Symmetry Groups in The Ornament

Twelve ornament units were classified across the three scales of symmetry and the curved surfaces of the complex. Table 1 summarises the classification; the subsections below present the mathematical concepts with annotated figures.

Table 1. Symmetry groups identified in the ceramic ornament of Masjid Tiban

No.	Ornament unit (zone)	Symmetry elements	Group (IUC; Conway)	|G| / level
1	Eight-ray glazed star (gate pillars, courtyard columns; Fig. 6)	Rotations by 45k°; 8 mirror lines	D_8	16 / rosette
2	Star tile within square frame (gate pillars; Fig. 6)	Rotations by 90k°; 4 mirror lines	D_4	8 / rosette
3	Pyramidal square relief tile (information hall; Fig. 2)	Rotations by 90k°; 4 mirror lines	D_4	8 / rosette
4	Square-meander (fret) tile (information hall; Fig. 10)	Rotations by 90k°; no mirrors (chiral)	C_4	4 / rosette
5	Whole gate façade (main gate; Fig. 1)	One vertical mirror	D_1	2 / rosette
6	Calligraphic panels (main gate; Fig. 1)	Identity only	C_1	1 / rosette
7	Vertical strip of star tiles with vine borders (gate pillars; Fig. 7), strip of relief tiles (hall; Fig. 2)	Translation; transverse and longitudinal mirrors; half-turns	p2mm (*22∞)	∞ / frieze
8	Arched gables and balustrade arcade (upper terraces; Figs. 4, 8)	Translation; transverse mirrors only	p1m1 (*∞∞)	∞ / frieze
9	Cartouche band with scroll infill (gate soffit, Fig. 9; hall cornice, Fig. 2)	Outline: translation and transverse mirrors; with infill: translation only	p1m1 → p1 (∞∞)	∞ / frieze
10	Field of fret tiles (information	Two translations; 90° and 180°	p4 (442)	∞ /

	hall; Fig. 10)	centres; no mirrors		wallpaper
11	Diagonal lattice screen (hall; Fig. 10) and diagonal blue–white chequer (façade; Fig. 3)	Two translations; 90° centres; mirrors in four directions	p4m (*442)	∞ / wallpaper
12	Tiled drum (gate; Fig. 1), tiered tower (Fig. 5), monumental vessel (Fig. 11)	Motif repeated n times about a vertical axis	C_n or D_n (frieze closed on a cylinder)	n or 2n / cyclic action

Source: Author's classification from field photographs, following Washburn and Crowe (1988)

Rosette level: The Eight-Ray Star and The Cost of a Frame

The most frequent motif on the gate pillars and courtyard columns is a glazed eight-ray star in relief, surrounded by eight white petals and a gold outline (Figure 6). Taken alone, the star is invariant under rotations by multiples of 45° and under reflection in eight mirror lines, four passing through opposite rays and four passing between them. Its symmetry group is the dihedral group of order 16, with the standard presentation (Figure 6 shows its mirror lines)

$$D_n = \langle r, s \mid r^n = s^2 = e,\ srs = r^{-1} \rangle,\ with\ n = 8\ and\ |D_8| = 2n = 16. \quad (1)$$



Figure 6. Mirror lines of the eight-ray star tile: solid lines are the four mirrors of D_4 kept by the square frame; dashed lines are the four further mirrors of D_8 removed by the frame

Every star, however, is set in a square tile. The square frame admits only rotations by multiples of 90° and four mirrors, so the symmetry of the complete tile is $D_4 = \langle r^2, s \rangle$, a subgroup of D_8. By Lagrange's theorem the index is $[D_8 : D_4] = 16/8 = 2$, so the frame removes exactly one coset, rD_4, namely the four rotations by odd multiples of 45° and the four mirrors lying between rays. The craft decision to place a star in a square tile is therefore, in algebraic terms, the restriction from a group to an index-2 subgroup.

The frame also changes how the group acts on the star's rays. Under D_8 the eight rays form a single orbit; the stabiliser of one ray contains the identity and the mirror through that ray, so the orbit–stabiliser theorem gives

$$|G| = |Orb(x)| \cdot |Stab(x)|:\ 16 = 8 \times 2\ under\ D_8,\ 8 = 4 \times 2\ under\ D_4. \quad (2)$$

Under D_4 the rays split into two orbits of size four: rays pointing to the corners of the tile and rays pointing to the midpoints of its edges. Visually, the eight identical rays become two classes of rays, a distinction created entirely by the frame.

Table 2 gives the Cayley table of D_4 for the framed tile, where r denotes the 90° anticlockwise rotation and s the reflection in the vertical axis. The table makes three structural properties visible: each element appears exactly once in every row and column; the rotations {e, r, r^2, r^3} form the cyclic subgroup C_4 in the upper-left block; and the group is non-abelian, since r

$\circ$ s = rs whereas s $\circ$ r = r^3s. On the tile, "rotate then reflect" and "reflect then rotate" send a marked ray to different positions.

Table 2. Cayley table of D_4, the symmetry group of the framed star tile (row $\circ$ column)

$\circ$	e	r	r^2	r^3	s	rs	r^2s	r^3s
e	e	r	r^2	r^3	s	rs	r^2s	r^3s
r	r	r^2	r^3	e	rs	r^2s	r^3s	s
r^2	r^2	r^3	e	r	r^2s	r^3s	s	rs
r^3	r^3	e	r	r^2	r^3s	s	rs	r^2s
s	s	r^3s	r^2s	rs	e	r^3	r^2	r
rs	rs	s	r^3s	r^2s	r	e	r^3	r^2
r^2s	r^2s	rs	s	r^3s	r^2	r	e	r^3
r^3s	r^3s	r^2s	rs	s	r^3	r^2	r	e

Source: Author's computation; r = rotation by 90°, s = reflection in the vertical axis

The other rosettes complete a subgroup chain. The pyramidal relief tiles on the information-hall wall (Figure 2) also have D_4 symmetry. The square-meander (fret) tiles of the same hall (Figure 2) turn in one direction only, so they keep the four rotations but lose every mirror: their group is C_4, the rotation subgroup of D_4. At the scale of the whole building element, the gate façade has a single vertical mirror (Figure 1), giving $D_1 \cong \mathbb{Z}_2$. The calligraphic panels at the top of the gate (Figure 1) have only the identity, C_1, because legible Arabic script cannot be reflected or rotated without loss of meaning. The gate therefore realises the chain $C_1 \leq D_1 \leq D_4 \leq D_8$, from the asymmetry of the sacred text to the eight-fold star.

Frieze Level: Bands, Borders, and Symmetry Breaking

The vertical strips of star tiles on the gate pillars, flanked by vine borders on both sides (Figure 7), are generated by a translation t along the pillar, a mirror transverse to the strip through each tile centre, and a mirror along the strip's central axis. Their composition yields half-turns about the tile centres, so the strip belongs to p2mm (*22∞), the most symmetric of the seven frieze groups. The arched gables and balustrade arcade of the upper terraces (Figures 4 and 8) are different: each arch is mirror-symmetric about its own vertical axis, but top and bottom differ, so there is no longitudinal mirror and no half-turn. Their group is p1m1 (*∞∞).



Figure 7. Pillar strip of star tiles as a p2mm frieze: translation t, transverse mirrors (dashed red and orange), longitudinal mirror (dashed cyan), and half-turn centres (diamonds)



Figure 8. Arched gables and tiled arcade as p1m1 friezes: translation t_1 and transverse mirrors of the gables (red) and of the arcade (orange)

The cartouche band on the gate soffit (Figure 9), repeated in the cornice of the information hall (Figure 2), shows symmetry breaking within a single band. The outline of each cartouche is mirror-symmetric, so the outlines alone form a p1m1 frieze. Each cartouche, however, is filled with a scroll that curls in one direction. The scroll is chiral, so the transverse mirrors are lost and only translations remain: the band as executed belongs to p1 (∞∞). Since p1 is the translation subgroup of p1m1 with index 2, the infill removes exactly half of the outline's symmetries. Together with the pillar strips, the bands realise the frieze chain p1 ≤ p1m1 ≤ p2mm.



Figure 9. Cartouche band on the gate soffit: mirror axes of the outlines (dashed, p1m1), translation t, and chiral scroll infill that reduces the band to p1

Wallpaper Level and Curved Surfaces

Two wallpaper groups appear. The field of square-meander tiles in the information hall (Figure 10) has two independent translations and centres of 90° rotation but no mirror or glide line, because each tile is chiral; its group is p4 (442). The diagonal lattice screen of the same hall (Figure 10) and the diagonal blue–white chequer of the residential façade (Figure 3) have, in addition, mirrors in four directions, giving p4m (*442). The relation p4 ≤ p4m, again of index 2, repeats at the wallpaper level the same distinction between chiral C_4 and achiral D_4 that appears at the rosette level.



Figure 10. Wallpaper groups in the information hall: diagonal lattice screen (p4m) and square-meander tiles (p4)

Where ornament wraps around a curved surface (the tiled drum above the gate, the tiers of the courtyard tower, and the bands of the monumental egg-shaped vessel; Figures 1, 5, and 11), a frieze is closed on itself. If the motif is repeated n times around the circumference, the infinite translation group $\mathbb{Z}$ of the flat band becomes the finite cyclic group $\mathbb{Z}/n\mathbb{Z}$ of rotations about the vertical axis. A closed p1 band therefore has symmetry C_n, and a closed p1m1 band has D_n (seen in plan). The curved surfaces thus illustrate a quotient construction, in which the same frieze generates a finite group whose order is set by how many times the artisans fitted the motif around the form.



Figure 11. Monumental vessel in the courtyard: ornamental bands (ellipses) closed around a vertical axis of rotation (dashed)

Counting the Design Space with Burnside's Lemma

Group actions also explain how much design freedom a motif allows. Suppose each of the eight rays of the star can be glazed in one of two colours, giving $2^8 = 256$ colourings. Two colourings represent the same design if a symmetry carries one onto the other. Burnside's lemma counts the distinct designs as the average number of colourings fixed by each group element:

$$|X/G| = (1/|G|)\,\Sigma\,|Fix(g)|, \quad g \in G. \qquad (3)$$

For the free-standing star, $G = D_8$. The identity fixes 256 colourings; the rotations by ±45° and ±135° fix 2 each, the rotations by ±90° fix 4 each, and the half-turn fixes 16; the four mirrors through rays fix 32 each and the four mirrors between rays fix 16 each. Hence $|X/D_8| = (256 + 8 + 8 + 16 + 128 + 64)/16 = 480/16 = 30$. For the framed star, $G = D_4$, whose four mirrors pass through rays: $|X/D_4| = (256 + 4 + 16 + 4 + 4 \times 32)/8 = 408/8 = 51$. Exhaustive enumeration of all 256 colourings confirms both counts. Framing the star thus increases the number of distinguishable two-colour designs from 30 to 51, because colourings that are equivalent on a free star become different once corner rays and edge rays are distinguished. The uniformly blue

star actually used is one fixed point of every symmetry, the colouring that preserves the full group.

Discussion

The findings show that the ornament of Masjid Tiban is organised as a multi-scale hierarchy of symmetry groups rather than as an assortment of shapes. Within one gate, the rosette chain $C_1 \leq D_1 \leq D_4 \leq D_8$ and the frieze chain p1 ≤ p1m1 ≤ p2mm coexist (Figures 1, 6, 7, and 9), and the wallpaper relation p4 ≤ p4m repeats the same index-2 logic in the information hall. This extends Sholikhah et al. (2025), who identified plane figures, solid figures, and individual transformations at the same site, by showing that those transformations combine into groups and that the groups nest into one another. The step matters mathematically, because a list of transformations becomes a structure only when closure, identity, inverses, and associativity are established, as Table 2 does for the framed tile.

Compared with Indonesian textile studies, the repertoire is narrower but more deliberately hierarchical. Indriati et al. (2022) found all seven frieze groups in Siak weaving, and the generative studies of Nataliani (2022), Mingka et al. (2023), and Sinaga and Kartika (2024) used the full frieze and wallpaper catalogues to produce new motifs. At Masjid Tiban, the identified groups concentrate on four-fold and eight-fold symmetry (D_4, D_8, p4, p4m) together with their index-2 subgroups. This concentration is consistent with the prominence of eight-fold stars in Islamic ornament (Lu & Steinhardt, 2007) and with the material logic of square ceramic tiles, which favour four-fold lattices. The present analysis therefore adds an architectural and ceramic case to a literature dominated by textiles.

Pedagogically, the hierarchy supports a coherent learning trajectory for introductory group theory, summarised in Table 3. It addresses known difficulties: students' fragile understanding of subgroups, cosets, and quotient structures (Dubinsky et al., 1994), and the gap between procedural and conceptual understanding documented by Melhuish (2019). In line with Larsen's (2013) guided reinvention, students begin by physically manipulating a cut-out of the framed star tile, record compositions, and construct Table 2 for themselves before the group axioms are named. The cultural anchor also responds to calls for ethnomathematics to move from recognition tasks to structured curricular integration (Lidinillah et al., 2022; Prahmana & D'Ambrosio, 2020; Verner et al., 2019).

Table 3. Proposed learning trajectory for introductory group theory anchored in Masjid Tiban ornament

Phase	Ornament anchor	Group-theory concept	Student task
Symmetry as action	Framed star tile (Table 1, no. 2)	Isometry, composition, group axioms	Find all motions that return a tile cut-out to its outline; build the Cayley table (Table 2).
Subgroups and index	Free star vs. framed star; fret tile	Subgroup, Lagrange's theorem, cosets	Explain which symmetries the frame removes; list the coset rD_4 and verify $[D_8 : D_4] = 2$.
Actions and orbits	Rays of the star	Group action, orbit–stabiliser theorem	Partition the rays into orbits under D_8 and D_4; verify Equation (2).
Infinite groups	Pillar strip, cartouche band, tile fields	Frieze and wallpaper groups, symmetry breaking	Classify bands with the flowchart; explain how the scroll infill turns p1m1 into p1.
Counting and quotients	Two-colour stars; drum and vessel bands	Burnside's lemma, quotient $\mathbb{Z} \to \mathbb{Z}/n\mathbb{Z}$	Count two-colour star designs (30 vs. 51); predict the group of a band wrapped n times.

Source: Author's design, adapted from the guided-reinvention approach of Larsen (2013)

The dialogic step of ethnomodelling (Rosa & Orey, 2016) brings the emic themes and the etic model together (Table 4). The absence of construction drawings, combined with phased work by many hands, explains why symmetry at Masjid Tiban is carried by modular units rather than by a global plan. A pattern built from identical tiles placed relative to their neighbours is, in algebraic terms, generated locally: a translation and a few reflections applied repeatedly produce the whole frieze or wallpaper, much as a group is produced from its generators. Such local rules allow coherence to survive interruptions, changes of workers, and decades of growth, which is exactly what the documentary record describes. The same groups (D_4, D_8, and p2mm) recur in the gate, the hall, and the courtyard, even though these zones were built at different times. Craft practice of this kind generates group structure as an implicit property of the work, in the sense of D'Ambrosio's (1985) mathematics in action, without requiring the builders to hold the formal concepts. The eclectic stylistic sources also map onto distinct subgroups: the eight-fold star (D_8) belongs to the Islamic geometric tradition (Lu & Steinhardt, 2007), the pinwheel fret (C_4, p4) resembles the meanders of Chinese ornament, and the scrolled cartouche (p1) recalls floral arabesque. Unity is supplied not by a single style but by the shared palette and the shared symmetry grammar. Finally, the documented prominence of Qur'anic calligraphy supports reading the trivial group C_1 of the text panels as a place where symmetry gives way to legible scripture; this reading is interpretive, since the sources do not record the founder's intention on this point.

Table 4. Dialogic correspondence between emic themes and the etic group-theoretic model

Emic theme (source)	Evidence in the ornament	Group-theoretic reading
Design without drawings, guided by istikharah (AW, 2014; Suryo Putro et al., 2023)	Patterns defined by tile units and their repetition, not by a global layout	Local generation: a few generators (t, reflections) applied repeatedly produce the frieze or wallpaper group
Manual construction by santri, alumni, and congregants (Sholah, 2018)	Identical moulded star tiles on the gate pillars and courtyard columns (Figures 1, 5, 6)	Congruent copies form the orbit of one motif under the translation subgroup
Phased, interrupted construction over decades (Hakim, 2016; Suryo Putro et al., 2023)	D_4, D_8, and p2mm recur in zones built at different times (Figures 1, 2, 5)	Invariance of the symmetry repertoire across construction phases
Blend of Middle Eastern, Chinese, and Indian forms under a blue-and-white palette (AW, 2014; Suryo Putro et al., 2023)	Eight-ray stars, pinwheel fret tiles, scrolled cartouches	Stylistic sources map onto distinct groups: D_8, C_4 / p4, and p1
Qur'anic calligraphy and Asmaul Husna (AW, 2014; Suryo Putro et al., 2023)	Text panels on the gate and walls (Figure 1)	Trivial group C_1: symmetry gives way to legible text (interpretive)

Source: Author's synthesis of documentary sources and the classification in Table 1

Several limitations should be noted. The classification relied on photographic documentation and written sources; the emic perspective is therefore mediated by published accounts rather than drawn from direct testimony of caretakers or artisans. Only five ornament zones of a large multi-storey complex were examined, and colour was treated as a fixed attribute rather than analysed through two-colour (counterchange) symmetry groups, which the blue–white palette invites. The learning trajectory in Table 3 is a design proposal and has not yet been implemented. Future research should combine a complete survey of the complex with interviews of community members, extend the analysis to colour symmetry, and test the trajectory with pre-service teachers in a design-research cycle.

CONCLUSION

This study set out to read the ceramic ornament of Masjid Tiban through one mathematical topic, group theory of symmetry, rather than through the catalogue of plane and solid figures that dominates ethnomathematics research. Twelve ornament units were found to embody the rosette groups C_1, D_1, C_4, D_4, and D_8; the frieze groups p1, p1m1, and p2mm; the wallpaper groups p4 and p4m; and cyclic actions on curved surfaces. These groups are not isolated: they form subgroup chains connected by index-2 restrictions, so framing, chirality, and calligraphy act as visible mechanisms of symmetry breaking. Lagrange's theorem, the orbit–stabiliser theorem, and Burnside's lemma each have a direct and verifiable counterpart in the ornament. Read together with documentary accounts of a complex designed without drawings and built by hand over decades, the findings show how local, modular craft practice can sustain a coherent global symmetry structure. Theoretically, the study shows that ethnomathematical analysis can reach the level of algebraic *structure* and not only that of *shape*, and that architectural ceramics deserve the same group-theoretic attention that Indonesian textiles have received. Practically, it provides a culturally grounded and mathematically rigorous anchor for teaching abstract algebra, together with a five-phase learning trajectory for pre-service teachers that also promotes appreciation of local Islamic heritage. Future work should complete the survey of the complex, gather emic accounts from caretakers and artisans, analyse two-colour symmetry in the blue–white palette, and evaluate the proposed trajectory empirically in teacher education classrooms.

DECLARATIONS

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Author Contributions Statement

Badiatus Sholikhah: Conceptualization, Methodology, Investigation, Formal analysis, Data Curation, Visualization, Writing – Original Draft, and Writing – Review & Editing. As the sole author, she is responsible for all aspects of the work.

AI Usage Statement

During the preparation of this manuscript, the author used an AI tool to assist with text drafting, preliminary symmetry classification, computer-based verification of counting results, and the preparation of annotated figures. The author independently reviewed all classifications against the field photographs, verified all references against their original sources, critically revised the generated content, and assumes full responsibility for the accuracy and integrity of the published work.

Conflict of Interest

The author declares that there are no conflicts of interest related to the research, authorship, and/or publication of this article.

Data Availability

The photographs and classification records supporting the findings of this study are available from the author upon reasonable request.

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Ethical Approval

This study analysed ornament on a publicly accessible religious building and published documentary sources and did not involve human participants; formal ethical approval was therefore not required. Photographs were taken from publicly accessible areas, and no individuals are identifiable.

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