Ethnomathica
Journal of Ethnomathematics Studies
E-ISSN: XXXX-XXXX
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VOLUME / ISSUE	PAGES	E-ISSN	DOI
Vol. 1, No. 1 (2026)	67-83	XXXX-XXXX	

Ethnomathematical Exploration of Functions, Logarithms, and Transcendental Equations in The Kecapi and Calong Traditional Musical Instruments of Mandar, West Sulawesi

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Received: March 24, 2026 **Revised:** Mei 08, 2026 **Accepted:** June 10, 2026 **Published:** June 30, 2026

ABSTRACT

Ethnomathematics research on Indonesian cultural artefacts has been dominated by the identification of plane and solid figures, leaving the functional, analytic, and acoustic mathematics embedded in artefacts largely unexplored. This study explores the mathematical ideas embodied in two traditional musical instruments of the Mandar people of West Sulawesi, the boat-shaped two-stringed kecapi and the coconut-resonated calong, deliberately excluding plane and solid geometry. A qualitative exploratory ethnomathematics design framed by ethnomodelling (emic, etic, and dialogic perspectives) was combined with digital mathematical modelling. Three photographs of the instruments were digitised, and the resulting data were modelled with least-squares regression, polar-coordinate analysis, logarithmic transformation, and numerical root finding, with every model made reproducible in GeoGebra. Five mathematical ideas emerged. The kecapi keel is described by a quadratic function (R^2 = 0.999) that is practically indistinguishable from a catenary, with maximum curvature at about one quarter of the body length. The carved tail scroll follows a logarithmic spiral (R^2 = 0.991) better than an Archimedean spiral. The five finger-posts are equally spaced (coefficient of variation 5.3%), forming an arithmetic sequence whose musical intervals follow a reciprocal-logarithmic function. The vibration of each calong key is governed by the transcendental equation cos x cosh x = 1, whose roots locate the ideal support points at 0.224 and 0.776 of the key length, and its pitch follows an inverse-square power function of key length. The findings extend ethnomathematics beyond shapes toward functions, sequences, logarithms, and hyperbolic functions, and offer culturally grounded, technology-supported contexts for secondary and tertiary mathematics and physics education.

Keywords: Ethnomathematics; Ethnomodelling; Mandar Musical Instruments; Mathematical Modelling; GeoGebra

How to Cite: Andini, A., Lela, N., & Arif, M. R. A. F. (2026). Ethnomathematical Exploration of Functions, Logarithms, and Transcendental Equations in The Kecapi and Calong Traditional Musical Instruments of Mandar, West Sulawesi. *Ethnomathica: Journal of Ethnomathematics Studies, 1(1), 67–83.*

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INTRODUCTION

Ethnomathematics emerged as a research programme that recognises mathematics as a set of culturally situated practices of counting, measuring, locating, designing, and explaining that are developed by identifiable cultural groups (D'Ambrosio, 1985; Barton, 1996). Over the past four decades, the programme has moved from documenting "hidden" mathematics towards using cultural knowledge as a legitimate entry point to school mathematics (Rosa & Orey, 2011).

Recent bibliometric analyses confirm that ethnomathematics has become a rapidly growing field, with Indonesia among the most productive contributors and with mathematics learning, students, and local culture as the most frequent keywords (Deda et al., 2024; Nasrum et al., 2025). The same analyses, however, identify geometry as one of the dominant clusters and call for studies that address other mathematical abilities and that integrate technology (Deda et al., 2024).

The dominance of geometry is particularly visible in Indonesian studies of cultural artefacts. A systematic review of Scopus-indexed work concluded that traditional houses, dances, foods, batik, woven fabrics, temples, and mosques are mostly read through the lens of geometric shapes (Iskandar et al., 2022), and even exemplary studies of batik focus on geometric patterns and transformations (Prahmana & D'Ambrosio, 2020). Studies of Sundanese practices of estimating, measuring, and making patterns show that richer ideas are available (Muhtadi et al., 2017), yet musical instruments, although intrinsically mathematical, have mostly been read in the same way. A review of ten Indonesian traditional instruments reported that their mathematical content was mainly framed as plane and solid figures and simple number patterns (Kristyasari & Sukoco, 2022), and a review of studies on the rebana frame drum found the same focus on circles, flat figures, solids, surface area, and volume (Nisa' & Malasari, 2024). Other studies have identified sets and numbers in Javanese gamelan (Oktaviyani et al., 2023), ratios and proportions in Sundanese gamelan making (Supriyadi et al., 2024), and arithmetic sequences and relations in the wooden gong of Rote Ndao (Panie et al., 2024). These studies show that instruments can carry richer mathematics, yet the analytic ideas that govern how an instrument is shaped and how it sounds, such as functions, logarithms, and differential equations, remain almost absent from ethnomathematics literature.

This gap is noteworthy because music is one of the oldest meeting points between mathematics and culture. Musical intervals are ratios of frequencies, perceived pitch grows logarithmically with frequency, and the vibration of strings and bars is described by differential equations whose solutions involve trigonometric and hyperbolic functions (Benson, 2006; Fletcher & Rossing, 1998). Classroom studies have shown that such connections allow fractions, sequences, exponents, and logarithms to be taught in meaningful contexts (Korhonen, 2022), and ethnoscience studies of Indonesian percussion have already used instruments as contexts for sound waves and frequency in physics learning (Sarah et al., 2024). What has not yet been done is to read these analytic ideas from the material form of a specific cultural instrument and to model them in a way that respects the cultural knowledge of its makers.

Ethnomodelling provides the methodological bridge for such a reading. It treats the mathematical modelling of cultural practices as a dialogue between emic knowledge, which is the insider's understanding of a practice, and etic knowledge, which is the outsider's academic mathematics, with a dialogic synthesis that neither reduces the cultural practice to school mathematics nor romanticises it (Orey & Rosa, 2021). Recent work has shown that this approach can link community practices to school mathematics in a principled way (Sudirman et al., 2024) and can activate rich mathematical connections in teacher education (Rodríguez-Nieto et al., 2025). Digital tools strengthen this bridge. GeoGebra has become the most widely researched dynamic mathematics software, valued for turning abstract ideas into manipulable representations (Awaji et al., 2025), and it supports modelling with functions and differential equations in secondary school (Latifi et al., 2022) as well as culture-based learning design (Mataheru et al., 2023).

The Mandar people of West Sulawesi possess two instruments that are well suited to this inquiry. The kecapi Mandar (kacaping) is a long, boat-shaped, two-stringed lute whose strings

are stopped against five finger-posts, played by skilled performers (pakkacaping) to accompany sung verse at weddings, circumcisions, and religious gatherings (Keen, 2016). It is now rare, transmitted within families, and its body is notably more curved than that of other Indonesian kecapi (Raodah, 2019). The calong is a small idiophone whose wooden or bamboo keys are laid over a hollowed coconut resonator; it originated among Mandar farmers as a signalling and leisure instrument and is now revived as an extracurricular instrument in schools (Raodah, 2019). To the best of the authors' knowledge, neither instrument has been examined from an ethnomathematical perspective.

Accordingly, this study aims to explore and model the non-geometric mathematical ideas embodied in the kecapi Mandar and the calong, deliberately excluding plane and solid figures. Three research questions guide the study: (1) which functional and analytic mathematical ideas can be identified in the form and sound production of the two instruments; (2) how can these ideas be modelled reproducibly with digitised data and GeoGebra; and (3) what contributions do the resulting models offer to mathematics and science education. The study argues that cultural musical instruments are not merely containers of shapes but embodiments of functions, sequences, logarithms, and even transcendental equations, and that recognising this expands the reach of ethnomathematics into the content of upper secondary and tertiary mathematics.

METHOD

The study followed a structured procedure combining ethnographic interpretation of cultural sources with digital mathematical modelling. The design, data sources, data collection, and data analysis are described below.

Research Design

This study employed a qualitative exploratory ethnomathematics design framed by ethnomodelling (Orey & Rosa, 2021). The emic dimension was constructed from published ethnographic and documentary accounts of how the instruments are made, named, and played (Keen, 2016; Raodah, 2019). The etic dimension consisted of mathematical models built from measurements digitised from photographs of the instruments. The dialogic dimension interpreted each model against its cultural and acoustic function and mapped it to curricular content. Quantitative procedures (regression and numerical analysis) were used only as tools for describing the artefacts, not for hypothesis testing; hence the study remains qualitative and exploratory in its logic of inquiry.

Artefacts and Sources

The objects of study were one calong and two specimens of kecapi Mandar documented in three photographs, presented as Figures 1–3 in the Results section. The calong photograph shows the keys, coconut resonator, support rails, and binding system from an oblique top view. The first kecapi photograph shows a dark-wood specimen from an oblique view in which the keel, the carved tail scroll, the strings, the central post, the five finger-posts, and the carved floral head are visible. The second kecapi photograph shows a lighter specimen in a near-side view that allows the finger-post spacing to be measured with less perspective distortion. Documentary sources comprised ethnographic and cultural studies of Mandar music and the acoustics literature on strings and vibrating bars. Because the study used published sources and photographs only, no human participants were involved.

Data Collection

Data were collected through documentary study and photographic digitisation. For each photograph, pixel coordinates were extracted by intensity thresholding of the dark wood against the light background. The keel of the dark kecapi was sampled every 20 pixels along the horizontal axis, giving 47 boundary points between the tail and the head. The tail scroll was sampled along 25 polar rays at 10° intervals from a centre chosen to minimise the residual of the spiral model. The finger-posts were located from the brightness peaks of their lit upper faces in both kecapi photographs, together with the string anchor at the tail (T) and the nut at the head (N). All coordinates were normalised by the relevant reference length so that the results are scale-free. Digitisation was scripted in Python for reproducibility, and every model was re-expressed as GeoGebra commands (Table 1) so that teachers and students can reproduce it by inserting the photograph as a background image.

Table 1. GeoGebra commands used to reproduce the models

Exploration	Mathematical idea	GeoGebra command(s)	Output
Keel profile	Quadratic function, derivative	L = {(u_1, v_1), …}; v = FitPoly(L, 2); v'(x)	v(u), vertex, slope
Keel profile	Curvature	Curvature((u, v(u)), v); Extremum(v, 0, 1)	κ(u), κ max
Tail scroll	Logarithmic spiral	P = {(θ_1, r_1), …}; s = FitExp(P); FitLine({(θ_i, ln(r_i))})	a, b, R^2
Tail scroll	Polar curve	Curve(a e^(b t) cos(t + π/2), a e^(b t) sin(t + π/2), t, 0, 4.2)	Overlay on photo
Finger-posts	Arithmetic sequence, logarithm	δ = Slider(0, 0.1); Sequence((k, -1200 log(2, 1 - k δ)), k, 0, 5)	c_k in cents
Calong key	Transcendental equation	Intersect(cos(x), 1/cosh(x), 4, 5); NSolve(cos(x) cosh(x) = 1, x = 7.8)	x_1, x_2, x_3
Calong key	Mode shape, nodes	W(s) = cosh(x_1 s) + cos(x_1 s) - σ (sinh(x_1 s) + sin(x_1 s)); Root(W, 0.1, 0.4)	s = 0.224
Calong key	Power function	f(L) = 1 / L^2; Sequence((2^(-c/2400), 2^(c/1200)), c, {0, 200, 400, 700})	Design ratios

Data Analysis Techniques

Analysis proceeded in four stages. First, emic description identified the parts of each instrument, their local names and functions, and the cultural knowledge associated with them. Second, etic modelling fitted candidate functions by least squares and compared them using the coefficient of determination (R^2) and the root-mean-square error (RMSE); the most parsimonious model with comparable fit was retained. Third, acoustic-mathematical modelling applied the established physics of strings and free–free bars (Fletcher & Rossing, 1998) to derive the functions and equations that govern pitch, and solved them numerically. Fourth, the dialogic stage interpreted each model in relation to its cultural function and mapped the mathematical ideas to curricular content and GeoGebra activities. Trustworthiness was supported by triangulating two photographs of the kecapi, by comparing competing models rather than fitting a single one, and by reporting the limitations of photographic measurement explicitly, in particular the effect of perspective on absolute lengths.

RESULTS AND DISCUSSION

Results

The results are organised in three parts: the cultural description of the instruments, the mathematical explorations of the kecapi Mandar (Explorations 1–3), and the mathematical

explorations of the calong (Explorations 4–5). Each exploration is shown in its own figure so that the digitised data, the model, and the mathematical interpretation can be read separately.

The Instruments in Their Cultural Context

Figure 1 shows the calong. Four flat wooden keys of slightly different lengths are laid side by side over a large coconut that has been hollowed and opened at the top to act as a resonator. The keys rest on two wooden rails bound with rope and are held in place by bent wire staples, so that each key can vibrate freely when struck with a short stick. The calong is said to have emerged among Mandar farmers, who used it to send signals and to rest from work in the fields; its sound is louder and higher than that of a marimba, and the coconut must be neither too young nor too old while the keys are dried for months before use (Raodah, 2019).



Figure 1. The Calong Mandar: Four Free Keys Over A Hollowed Coconut Resonator

Figures 2 and 3 show two specimens of the kecapi Mandar. The body is carved from a single piece of wood in the form of a boat (lopi), rising at the head into a carved floral panel and ending at the tail in a rolled scroll. Two strings run from the tail, past a central post, to tuning pegs below the carved head, and the player stops them against five raised finger-posts near the head. The instrument accompanies sung verses that may be religious (masaqala), historical (toloq), or improvised and humorous (tedhe), and women performers use slightly modified instruments and techniques (Keen, 2016). The boat form is culturally meaningful for a maritime society whose identity is closely tied to seafaring, and the curvature of the body is described as more pronounced than in other kecapi traditions (Raodah, 2019).



Figure 2. Kecapi Mandar (Dark-Wood Specimen), Oblique View Showing Keel, Tail Scroll, Strings, Finger-Posts, and Carved Head



Figure 3. Kecapi Mandar (Light-Wood Specimen), Near-Side View Showing The Five Finger-Posts And The Openwork Head

Table 2 summarises the dialogue between the emic description of each component and the etic mathematical ideas explored in the rest of this section. None of these ideas belongs to the domain of plane or solid figures.

Table 2. Instrument Components, Cultural Meaning, and The Mathematical Ideas Explored

Component	Emic description (cultural knowledge)	Etic mathematical idea	Exploration
Kecapi keel (boat-shaped body)	Body carved as a boat (lopi); strongly curved; resonating chamber	Quadratic and catenary functions; derivative; curvature	1
Kecapi tail scroll	Rolled ornament at the tail where the strings are bound	Polar curve; logarithmic spiral; logarithmic linearisation	2
Kecapi finger-posts	Five raised posts against which the strings are stopped	Arithmetic sequence; reciprocal function; logarithm (cents)	3
Calong keys and rails	Keys rest on rope-bound rails and vibrate freely when struck	Transcendental equation; hyperbolic functions; nodes	4
Calong key lengths	Keys of different lengths give different pitches	Power (inverse-square) function; exponents	5

Source: Authors' analysis based on Keen (2016) and Raodah (2019)

Exploration 1: The Keel of the Kecapi as a Quadratic Function

The lower edge of the boat-shaped body (the keel) was digitised from Figure 2 between the end of the tail scroll (point A) and the base of the head (point B), as shown in Figure 4.

Coordinates were normalised by the horizontal length L of the segment AB, so that u = x/L runs from 0 to 1 and v = y/L measures the height of the keel above its lowest point.



Figure 4. Digitisation ff The Kecapi Keel: 47 Boundary Points From Tail (A) To Head (B) With The Fitted Model Superimposed

Four candidate models were compared: polynomials of degree 2, 3, and 4, and a catenary. Table 3 shows that all models explain more than 99.8% of the variance and that increasing the degree beyond two yields only marginal improvement. The quadratic function was therefore retained as the most parsimonious model:

$$v(u) = au^2 + bu + c, \quad u = \frac{x}{L},\ v = \frac{y}{L} \tag{1}$$

with a = 0.7486, b = −0.3697, and c = 0.0505 (Figure 5). The catenary v = a cosh((u − h)/a) + k, the curve of a hanging chain, fits equally well (R^2 = 0.9986) and is visually indistinguishable from the parabola over the observed range.

Table 3. Comparison Of Function Models For The Kecapi Keel (n = 47, Normalised Coordinates)

Model	Equation	R^2	RMSE
Quadratic	$v = 0.7486u^2 - 0.3697u + 0.0505$	0.9987	0.0046
Cubic	$v = 0.0804u^3 + 0.6281u^2 - 0.3220u + 0.0467$	0.9988	0.0043
Quartic	$v = -0.3903u^4 + 0.8610u^3 + 0.1297u^2 - 0.2140u + 0.0418$	0.9991	0.0038
Catenary	$v = 0.7364 \cosh((u - 0.2395)/0.7364) - 0.7294$	0.9986	0.0046

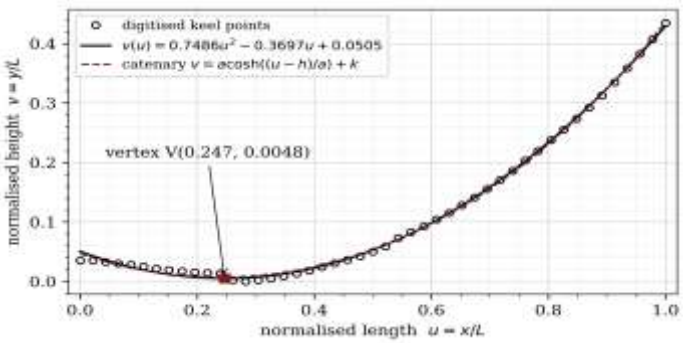

Figure 5. Quadratic and Catenary Models Of The Kecapi Keel With The Vertex V

The derivative v′(u) = 1.4973u − 0.3697 vanishes at u = 0.247, so the lowest point of the keel lies at about one quarter of the body length from the tail, below the region where the player rests the instrument. The slope changes from −0.37 at the tail to 1.13 at the head, which corresponds to a rise angle of about 48° where the body sweeps up into the head. The curvature was computed with

$$\kappa(u) = \frac{|v''(u)|}{(1+[v'(u)]^2)^{3/2}} \quad (2)$$

and reaches its maximum κ = 1.497 exactly at the vertex, then decreases steadily towards the head (Figure 6). Because the second derivative is constant and positive, the keel has no inflection point: it is a single convex sweep, bending most strongly where the body is deepest and straightening gradually as it rises.

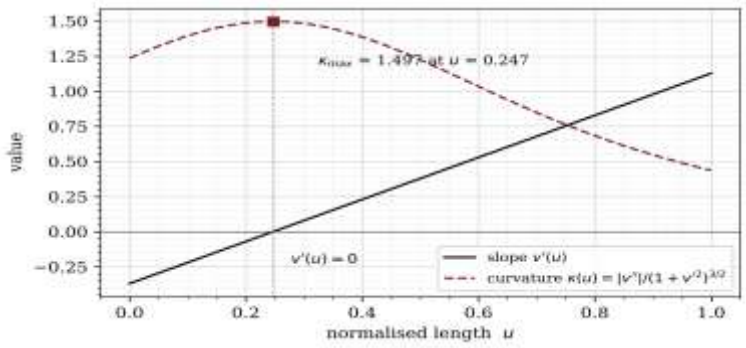

Figure 6. Slope v′(u) and Curvature κ(u) of the Keel Model

Exploration 2: The Tail Scroll as a Logarithmic Spiral

The tail of the kecapi ends in a rolled scroll around which the strings are bound. Its outer edge was digitised along 25 polar rays from a centre O, starting at the top of the scroll (θ = 0) and winding through 240° (Figure 7). The radius decreases continuously as the curve winds inward, which suggests a spiral. Two classical spirals were tested, the logarithmic spiral and the Archimedean spiral:

$$r(\theta) = a\, e^{b\theta} \quad (3)$$

$$r(\theta) = a + b\,\theta \quad (4)$$



Figure 7. Polar digitisation of the tail scroll: 25 rays from centre O and the fitted logarithmic spiral

Taking logarithms of Equation (3) transforms the spiral into a linear relation between ln r and θ,

$$\ln r = \ln a + b\,\theta \quad (5)$$

so that the growth parameter b becomes the slope of a straight line. This linearisation is a direct application of the laws of logarithms and is shown in Figure 9. Table 4 and Figure 8 show that the logarithmic spiral fits the scroll better than the Archimedean spiral, with a higher R^2 and a smaller error, and that the Archimedean model systematically departs from the data at both ends of the curve.

Table 4. Comparison of spiral models for the tail scroll (n = 25 rays, 0 ≤ θ ≤ 4.19 rad)

Model	Fitted equation (r in pixels)	R^2	RMSE (px)
Logarithmic spiral	r = 133.69 e^(−0.193θ)	0.991	2.06
Archimedean spiral	r = 128.55 − 17.49θ	0.984	2.79

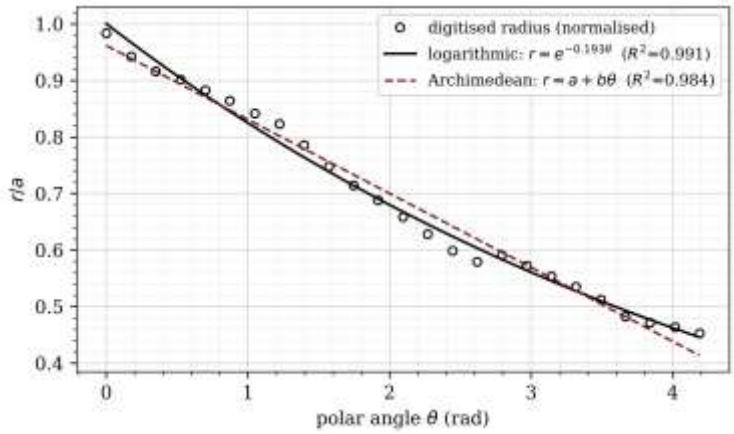

Figure 8. Radius Of The Tail Scroll Against Polar Angle With Logarithmic And Archimedean Models

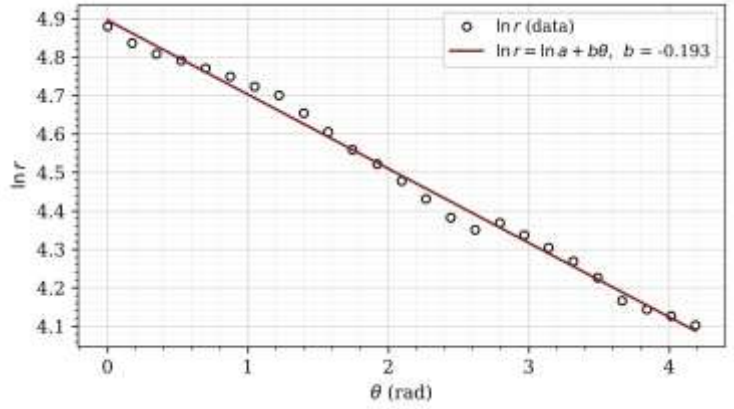

Figure 9. Logarithmic linearisation of the scroll: ln r is a linear function of θ

The fitted growth parameter b = −0.193 has a clear geometric meaning that does not depend on the size of the instrument. Each quarter turn multiplies the radius by e^(−0.193·π/2) = 0.739, so the scroll tightens by about 26% every 90°, and the angle between the tangent and the radius is constant at arctan(1/|b|) ≈ 79°. For comparison, the so-called golden spiral has |b| = 0.306, so the Mandar scroll is a gentler, more open logarithmic spiral.

Exploration 3: The Finger-posts as an Arithmetic Sequence and a Logarithmic Scale

The five finger-posts (P1–P5) and the nut (N) were located in both kecapi photographs, and the string anchor at the tail (T) defined the open-string length L_0 (Figure 10). In the near-side photograph (Figure 3), the distances between consecutive posts were all 16 ± 1 pixels for L_0 ≈ 416 pixels. In the oblique photograph (Figure 2), the five gaps measured along the neck were 24.8, 22.5, 23.9, 22.5, and 25.2 pixels (mean 23.8, standard deviation 1.27, coefficient of variation 5.3%). In both specimens, therefore, the posts are practically equally spaced: their positions form an arithmetic sequence,

$$L_k = L_0 - k\,d, \qquad \delta = \frac{d}{L_0}, \qquad k = 0,1,\ldots,5 \tag{6}$$

where d is the common difference and δ is the relative spacing. The estimated values are δ = 0.0385 for Figure 3 and δ = 0.0233 for Figure 2; the smaller value in the oblique view is consistent with the foreshortening of the head, which lies farther from the camera than the tail.

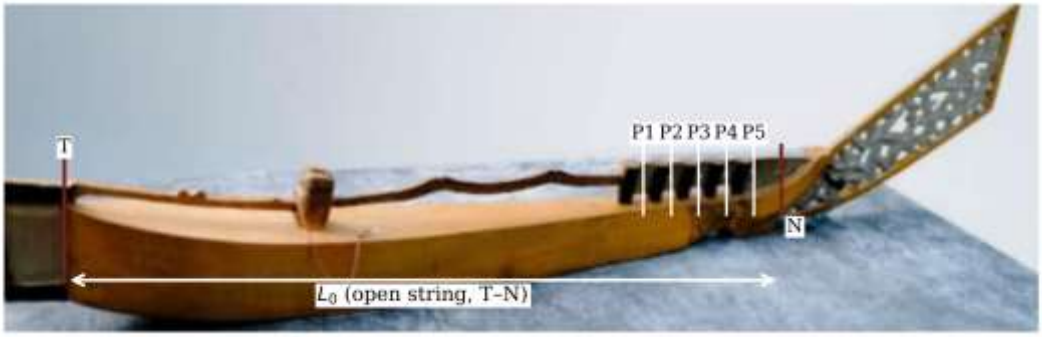

Figure 10. Positions of the string anchor (T), the five finger-posts (P1–P5), and the nut (N) on the kecapi

The musical meaning of this sequence follows from Mersenne's law for an ideal string of tension T and linear density μ (Fletcher & Rossing, 1998):

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \quad \Rightarrow \quad \frac{f_k}{f_0} = \frac{L_0}{L_k} \tag{7}$$

Pressing the string against post k shortens it from L_0 to L_k, so the frequency ratio is the reciprocal function $1/(1 - k\delta)$. Musicians, however, perceive intervals logarithmically. Expressed in cents, where one octave equals 1200 cents (Benson, 2006), the interval produced at post k is

$$c_k = 1200\log_2\frac{f_k}{f_0} = -1200\log_2(1 - k\delta) \tag{8}$$

Table 5 and Figure 11 show the resulting intervals. Because equal spacing in length does not produce equal steps in pitch, the successive intervals grow slowly from about 68 to 81 cents for δ = 0.0385. Expanding Equation (8) as a power series explains why the growth is so modest:

$$c_k = \frac{1200}{\ln 2}\left(k\delta + \frac{(k\delta)^2}{2} + \frac{(k\delta)^3}{3} + \cdots\right) \tag{9}$$

For small kδ the first term dominates, so an arithmetic sequence of posts produces an almost arithmetic sequence of pitches; the deviation from a perfectly equal-step (geometric) design appears only in the second-order term and amounts to about 30 cents at the fifth post. The equal spacing of the posts is thus a practical maker's rule whose acoustic error remains small because the posts occupy only a short portion of the string.

Table 5. Finger-post intervals implied by equal spacing (Equation 8)

Post k	Ratio f_k/f_0 (δ = 0.0233)	c_k (cents), δ = 0.0233	Ratio f_k/f_0 (δ = 0.0385)	c_k (cents), δ = 0.0385	Step (cents), δ = 0.0385
0 (open)	1.000	0.0	1.000	0.0	–
1	1.024	40.8	1.040	67.9	67.9
2	1.049	82.5	1.083	138.6	70.7
3	1.075	125.3	1.131	212.3	73.7
4	1.103	169.2	1.182	289.2	77.0
5	1.132	214.2	1.238	369.7	80.5

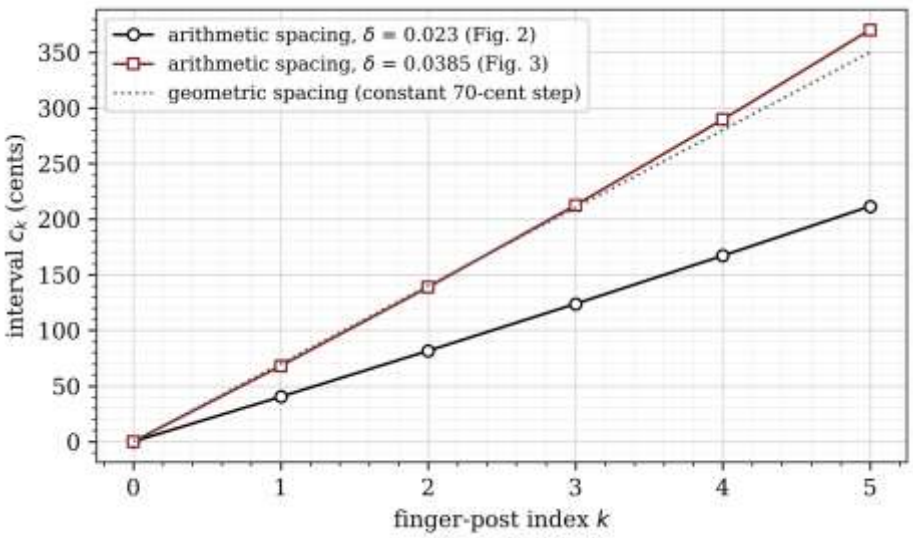

Figure 11. Intervals Produced By Arithmetically Spaced Finger-Posts Compared With A Geometric (Equal-Step) Design

Exploration 4: The Calong Key and the Transcendental Equation cos x cosh x = 1

Unlike a string, a calong key is a stiff bar with both ends free. Its transverse vibration w(x, t) is governed by the Euler–Bernoulli beam equation with zero bending moment and zero shear force at both ends (Fletcher & Rossing, 1998):

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} = 0, \qquad w''(0) = w'''(0) = w''(L) = w'''(L) = 0 \tag{10}$$

Separating variables and applying the four boundary conditions leads to the frequency equation

$$\cos(\beta L)\cosh(\beta L) = 1 \tag{11}$$

which is transcendental: it mixes a periodic trigonometric function with an exponentially growing hyperbolic function and cannot be solved algebraically. Rewriting it as cos x = 1/cosh x turns the problem into finding the intersections of two graphs, a task that GeoGebra performs with the Intersect command (Figure 12). The first three non-trivial roots are x_1 = 4.7300, x_2 = 7.8532, and x_3 = 10.9956. Because 1/cosh x decays rapidly, the roots quickly approach the zeros of cos x, that is, (2n + 1)π/2, which gives students an elegant approximation to check their numerical answers.

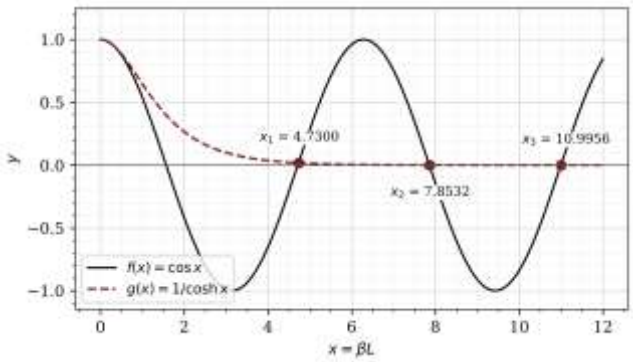

Figure 12. Graphical solution of cos x cosh x = 1 as the intersections of f(x) = cos x and g(x) = 1/cosh x

Substituting x_1 into the general solution gives the shape of the fundamental mode (Figure 13):

$$W(s) = \cosh(x_1 s) + \cos(x_1 s) - \sigma[\sinh(x_1 s) + \sin(x_1 s)], \quad \sigma = \frac{\cosh x_1 - \cos x_1}{\sinh x_1 - \sin x_1} \tag{12}$$

with σ = 0.9825. The function W(s) has two roots, at s = 0.224 and s = 0.776. These are the nodal points, where the key does not move when it vibrates in its fundamental mode. A key supported exactly at these points rings longest, because the supports absorb almost no energy. The calong keys in Figure 1 rest on rope-bound rails placed away from their ends rather than being fixed at the ends, which is the arrangement predicted by the model. Because the photograph is oblique, this correspondence is interpreted qualitatively, in terms of the support principle rather than exact rail coordinates.

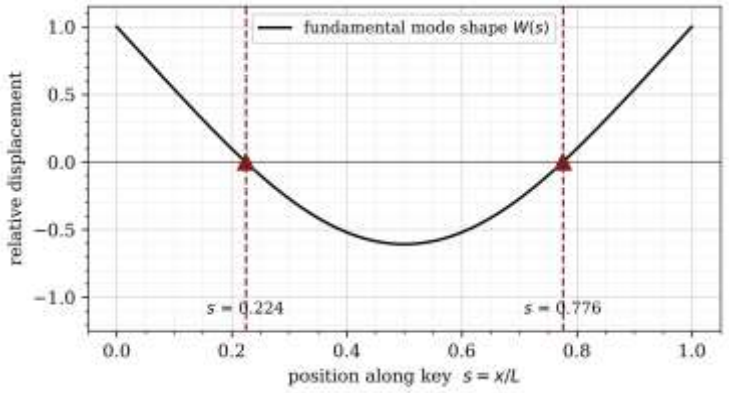

Figure 13. Fundamental mode shape of a free–free calong key with nodal points at s = 0.224 and s = 0.776

Exploration 5: Key Length and Pitch as a Power Function

The same analysis gives the frequencies of the key. For a key of length L, thickness h, Young's modulus E, and density ρ, the n-th frequency is

$$f_n = \frac{x_n^2}{2\pi L^2}\sqrt{\frac{EI}{\rho A}} = \frac{x_n^2}{2\pi L^2}\, h\sqrt{\frac{E}{12\rho}} \tag{13}$$

Two mathematical consequences follow. First, the overtone ratios $f_2/f_1 = (x_2/x_1)^2 = 2.757$ and $f_3/f_1 = 5.404$ are not integers, so the calong sound is inharmonic, which explains its bright, percussive character in contrast to the harmonic sound of the kecapi string. Second, for keys of the same wood and thickness, frequency is an inverse-square power function of length. Combining this with the logarithmic measure of intervals gives the design rule

$$\frac{f_k}{f_1} = \left(\frac{L_1}{L_k}\right)^2 \quad \Leftrightarrow \quad \frac{L_k}{L_1} = 2^{-c_k/2400} \tag{14}$$

Table 6 applies Equation (14) to two four-key tunings: an anhemitonic set and a diatonic tetrachord, the latter corresponding to the diatonic calong versions now made for school ensembles (Raodah, 2019). Figure 14 shows that a 700-cent interval (a fifth) requires the key to be shortened only to 0.817 of the reference length, because pitch depends on the square of length. In practice, makers can also raise pitch by thinning a key, since Equation (13) shows that frequency is directly proportional to thickness (Bork, 1995).

Table 6. Relative key lengths required by the power-function model (uniform keys, Equation 14)

Key	Anhemitonic set: c_k (cents)	f_k/f_1	L_k/L_1	Diatonic tetrachord: c_k (cents)	f_k/f_1	L_k/L_1
K1	0	1.000	1.000	0	1.000	1.000
K2	200	1.122	0.944	200	1.122	0.944
K3	400	1.260	0.891	400	1.260	0.891
K4	700	1.498	0.817	500	1.335	0.866

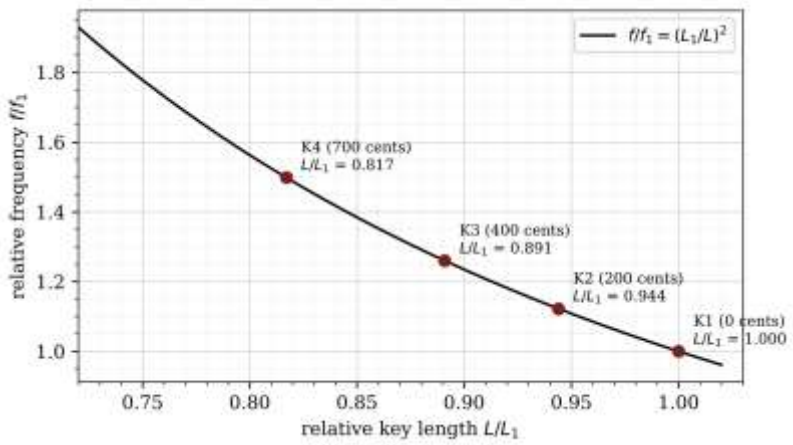

Figure 14. Inverse-square power function linking calong key length and frequency, with design points K1–K4

Table 7 synthesises the five explorations into learning opportunities. Each mathematical idea is connected to a feature of the instrument, an appropriate educational level, and a GeoGebra activity based on Table 1.

Table 7. Mapping of the Mathematical Ideas To Learning Contexts

Mathematical idea	Instrument feature	Educational level	Suggested GeoGebra activity
Quadratic function, vertex, derivative	Kecapi keel	Grades 10–11; first-year calculus	Digitise the keel on a photograph and fit FitPoly(L, 2); interpret the vertex
Curvature; catenary vs parabola	Kecapi keel	Calculus; teacher education	Compare Curvature() of the parabola and the catenary along the keel
Polar curves; exponential and logarithmic functions	Tail scroll	Grade 11–12; pre-calculus	Fit FitExp to (θ, r) data; linearise with ln r
Arithmetic sequences; reciprocal and logarithmic functions	Finger-posts	Grades 10–12	Use a slider for δ and plot cents with Sequence()
Transcendental equations; hyperbolic functions; numerical roots	Calong key	Tertiary mathematics and physics	Solve cos x cosh x = 1 by Intersect() and NSolve(); locate nodes
Power functions; exponents	Calong key lengths	Grades 10–12; physics of sound	Plot $f = 1/L^2$ and design four keys for a chosen scale

Discussion

The first contribution of this study is to show that two modest Mandar instruments embody a sequence of mathematical ideas that extends well beyond geometric shapes: quadratic and hyperbolic-cosine functions, derivatives and curvature, polar and logarithmic spirals, arithmetic sequences and logarithmic scales, power functions, and a transcendental equation. This responds directly to the observation that ethnomathematics research, and especially research on Indonesian artefacts and instruments, has concentrated on geometry and simple number patterns (Deda et al., 2024; Iskandar et al., 2022; Kristyasari & Sukoco, 2022; Nisa' & Malasari, 2024). Earlier studies of gamelan identified sets, numbers, and proportions (Oktaviyani et al., 2023; Supriyadi et al., 2024), and studies of wooden gongs identified arithmetic sequences (Panie et al., 2024). The present study goes a step further by moving from what the instrument looks like to how its form and sound are governed, which brings functional and analytic mathematics into the scope of ethnomathematics.

The keel result illustrates the dialogic character of ethnomodelling (Orey & Rosa, 2021). From an etic perspective, the parabola and the catenary are different functions with different physical meanings, yet over the observed range they are indistinguishable (R^2 = 0.9987 and 0.9986). From an emic perspective, the maker does not choose between them but bends and carves the wood until the body has the desired depth and sweep. The mathematics therefore describes, rather than prescribes, the craft. The location of the maximum curvature at about one quarter of the length is a quantitative trace of this craft knowledge, placing the deepest part of the resonating chamber below the region where the strings vibrate most freely. Such a finding can open classroom discussions on why different functions can model the same artefact and on how model choice depends on purpose, a central idea in mathematical modelling.

The logarithmic spiral of the tail scroll connects the kecapi to a family of forms that appear widely in nature and art because they look the same at every scale. The scroll's constant tangent–radius angle of about 79° and its 26% reduction in radius per quarter turn are scale-free descriptors that can be compared across specimens and workshops in future studies. Pedagogically, the linearisation of the spiral (Figure 9) offers an authentic reason to learn the laws of logarithms: a curve that is hard to fit by eye becomes a straight line after a single logarithmic transformation.

The finger-post result is perhaps the most culturally revealing. Western fretted instruments place frets in a geometric progression so that every step has the same musical size (Benson, 2006). The Mandar makers instead space the posts equally, a rule that is easy to carry out with a simple measuring stick. Equation (9) shows that this rule is not an error but a sound approximation: because the posts occupy only a short portion of the string, the arithmetic spacing deviates from equal steps only in the second-order term. The kecapi is played mainly as a drone with melodic ornaments over a few posts (Keen, 2016), a practice for which this degree of precision is musically adequate. This is a clear case of what Barton (1996) described as mathematics that makes sense within its own system of meaning. It also gives teachers a rich problem connecting arithmetic sequences, reciprocal functions, logarithms, and power series in one cultural context.

The calong explorations bring ethnomathematics into territory that is rarely visited: differential equations, hyperbolic functions, and transcendental equations. The rope-bound rails that hold the calong keys away from their ends reflect the same principle that guides the support of marimba and xylophone bars at their nodal lines (Bork, 1995; Fletcher & Rossing, 1998). The equation $\cos x \cosh x = 1$ is an ideal problem for showing students why numerical and graphical methods are needed, and GeoGebra makes the graphical solution immediate. This responds to calls for ethnomathematics research that integrates technology and develops higher mathematical abilities (Deda et al., 2024; Nasrum et al., 2025) and is consistent with evidence that GeoGebra supports conceptual understanding and modelling (Awaji et al., 2025; Latifi et al., 2022). The inverse-square relation between key length and pitch, combined with the logarithmic cent scale, also gives a concrete design task in which students can plan the keys of their own calong, linking mathematics with the physics of sound in the spirit of ethnoscience studies of Indonesian instruments (Sarah et al., 2024).

For mathematics education, the five explorations provide contexts for content that is usually taught without any cultural anchor in upper secondary and tertiary curricula (Table 7). For science education, they connect the same artefacts to standing waves, resonance, and vibrating bars, supporting interdisciplinary learning that respects local knowledge. For cultural preservation, the models provide quantitative, scale-free descriptors of instruments that are becoming rare (Raodah, 2019), which can complement ethnographic documentation.

The study has several limitations. First, the data were derived from photographs; although relative measures such as ratios, spacing patterns, and spiral parameters are robust to scale, perspective distortion affects absolute lengths, as shown by the difference between the two estimates of δ. Second, the acoustic results are model-based; the actual pitches of the kecapi and calong were not recorded. Third, the emic dimension relied on published ethnographic sources rather than direct interviews with makers and players. Future research should measure instruments directly, record and analyse their sounds spectrally, interview makers about the rules they use for curving the body, spacing the posts, and cutting the keys, and test the learning designs in Table 7 in classrooms.

CONCLUSION

This study set out to explore the non-geometric mathematical ideas embodied in the kecapi Mandar and the calong of West Sulawesi. Using an ethnomodelling design with digitised photographs and reproducible GeoGebra models, it identified five ideas: the keel of the kecapi follows a quadratic function practically identical to a catenary, with maximum curvature at one quarter of its length; the tail scroll is a logarithmic spiral; the finger-posts form an arithmetic sequence whose musical intervals follow a reciprocal-logarithmic function; the vibration of the calong key is governed by the transcendental equation $\cos x \cosh x = 1$, whose roots locate the nodal support points at 0.224 and 0.776 of the key length; and pitch is an inverse-square power function of key length. Theoretically, the findings show that ethnomathematics can move beyond the identification of shapes towards the functions, sequences, logarithms, and differential equations that govern how cultural artefacts are made and how they work. Practically, the models and GeoGebra commands provide ready-to-use, culturally grounded contexts for teaching functions, calculus, and the physics of sound, and quantitative descriptors that can support the documentation of endangered Mandar instruments. Future studies should combine direct measurement, acoustic recording, and interviews with makers and players to validate the models and to test their effectiveness in mathematics and science classrooms.

DECLARATIONS

Acknowledgments

The authors thank the Mandar makers and players whose knowledge sustains the kecapi and calong traditions, as well as the researchers and cultural documentarians whose published work made this study possible.

Author Contributions Statement

Andini: Conceptualization, Methodology, Formal Analysis, Visualization, Writing – Original Draft; Nurlela: Validation, Resources, Writing – Review & Editing.

AI Usage Statement

During the preparation of this manuscript, the authors used AI-assisted tools to support language drafting, Python scripts to digitise photographs and perform regression and numerical modelling, and software tools to render figures. All data, analyses, and references were carefully verified, and the models were independently rerun in GeoGebra. The authors take full responsibility for the accuracy and integrity of the manuscript's content.

Conflict of Interest

The authors declare that they have no conflicts of interest related to the research, authorship, and/or publication of this article.

Data Availability

The digitised coordinates, analysis scripts, and GeoGebra files supporting the findings of this study are available from the corresponding author upon reasonable request.

Funding

This research received no external funding.

Ethical Approval

This study analysed published documents and photographs of musical instruments and did not involve human participants; therefore, ethical approval was not required.

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